



General Certificate of Education
Advanced Level Examination
January 2013

Mathematics

MM2B

Unit Mechanics 2B

Monday 28 January 2013 9.00 am to 10.30 am

For this paper you must have:

- the blue AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed

- 1 hour 30 minutes

Instructions

- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.
- The **final** answer to questions requiring the use of calculators should be given to three significant figures, unless stated otherwise.
- Take $g = 9.8 \text{ m s}^{-2}$, unless stated otherwise.

Information

- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice

- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

- 1** Tim is playing cricket. He hits a ball at a point A . The speed of the ball immediately after being hit is 11 m s^{-1} .

The ball strikes a tree at a point B . The height of B is 5 metres above the height of A .

The ball is to be modelled as a particle of mass 0.16 kg being acted upon only by gravity.

- (a) Calculate the initial kinetic energy of the ball. (2 marks)
- (b) Calculate the potential energy gained by the ball as it moves from the point A to the point B . (2 marks)
- (c) (i) Find the kinetic energy of the ball immediately before it strikes the tree. (2 marks)
- (ii) Hence find the speed of the ball immediately before it strikes the tree. (2 marks)
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- 2** A particle moves in a horizontal plane. The vectors \mathbf{i} and \mathbf{j} are perpendicular unit vectors in the horizontal plane. At time t seconds, the velocity of the particle, $\mathbf{v} \text{ m s}^{-1}$, is given by

$$\mathbf{v} = 12 \cos\left(\frac{\pi}{3}t\right)\mathbf{i} - 9t^2\mathbf{j}$$

- (a) Find an expression for the acceleration of the particle at time t . (2 marks)
- (b) The particle, which has mass 4 kg , moves under the action of a single force, \mathbf{F} newtons.
- (i) Find an expression for the force \mathbf{F} in terms of t . (2 marks)
- (ii) Find the magnitude of \mathbf{F} when $t = 3$. (2 marks)
- (c) When $t = 3$, the particle is at the point with position vector $(4\mathbf{i} - 2\mathbf{j}) \text{ m}$.
Find the position vector, \mathbf{r} metres, of the particle at time t . (5 marks)
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- 3** A van, of mass 1500 kg , travels at a constant speed of 22 m s^{-1} up a slope inclined at an angle θ to the horizontal, where $\sin \theta = \frac{1}{25}$.

The van experiences a resistance force of 8000 N .

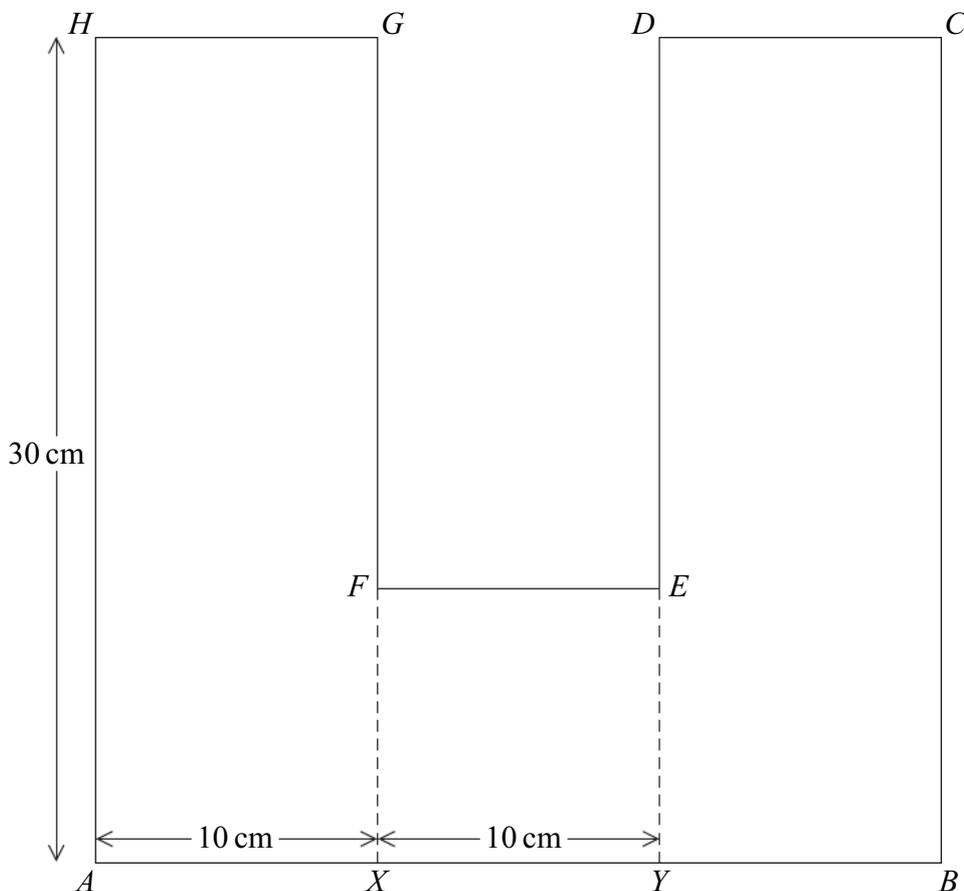
Find the power output of the van's engine, giving your answer in kilowatts.

(5 marks)



- 4 The diagram shows a uniform lamina which is in the shape of two identical rectangles $AXGH$ and $YBCD$ and a square $XYEF$, arranged as shown.

The length of AX is 10 cm, the length of XY is 10 cm and the length of AH is 30 cm.



- (a) Explain why the centre of mass of the lamina is 15 cm from AH . (1 mark)
- (b) Find the distance of the centre of mass of the lamina from AB . (3 marks)
- (c) The lamina is freely suspended from the point H .

Find, to the nearest degree, the angle between HG and the horizontal when the lamina is in equilibrium. (4 marks)

Turn over ►



- 5 A particle, of mass 12 kg, is moving along a straight horizontal line. At time t seconds, the particle has speed $v \text{ m s}^{-1}$. As the particle moves, it experiences a resistance force of magnitude $4v^{\frac{1}{3}}$. No other horizontal force acts on the particle.

The initial speed of the particle is 8 m s^{-1} .

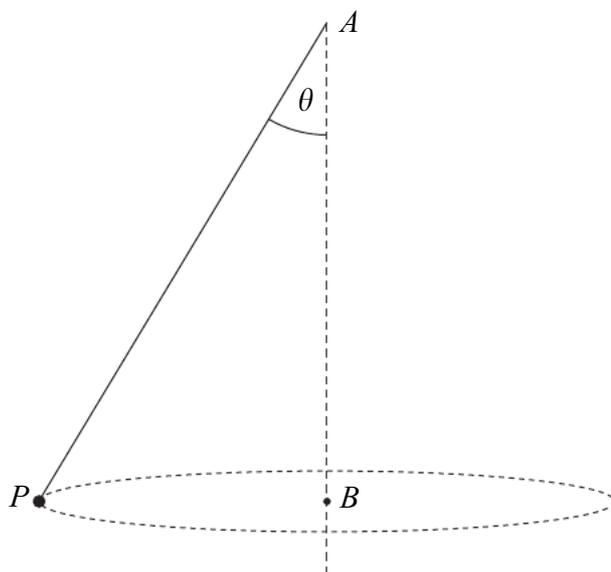
- (a) Show that

$$v = \left(4 - \frac{2}{9}t\right)^{\frac{3}{2}} \quad (6 \text{ marks})$$

- (b) Find the value of t when the particle comes to rest. (1 mark)

- 6 A light inextensible string has one end attached to a particle, P , of mass 2 kg. The other end of the string is attached to the fixed point A . The point A is vertically above the point B . The particle moves at a constant speed in a horizontal circle of radius 0.8 m and centre B . The tension in the string is 34 N.

The string is inclined at an angle θ to the vertical, as shown in the diagram.

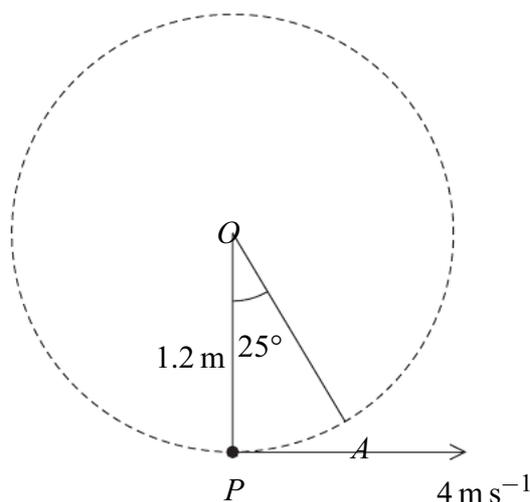


- (a) Find the angle θ . (3 marks)
- (b) Find the speed of the particle. (3 marks)
- (c) Find the time taken for the particle to make one complete revolution. (2 marks)



- 7 A small ball, of mass 3 kg, is suspended from a fixed point O by a light inextensible string of length 1.2 m. Initially, the string is taut and the ball is at the point P , vertically below O . The ball is then set into motion with an initial horizontal velocity of 4 m s^{-1} .

The ball moves in a vertical circle, centre O . The point A , on the circle, is such that angle AOP is 25° , as shown in the diagram.



- (a) Find the speed of the ball at the point A . (4 marks)
- (b) Find the tension in the string when the ball is at the point A . (3 marks)

- 8 (a) An elastic string has natural length l and modulus of elasticity λ . The string is stretched from length l to length $l + e$.

Show, by integration, that the work done in stretching the string is $\frac{\lambda e^2}{2l}$. (3 marks)

- (b) A particle, of mass 5 kg, is attached to one end of a light elastic string. The other end of the string is attached to a fixed point O .

The string has natural length 1.6 m and modulus of elasticity 392 N.

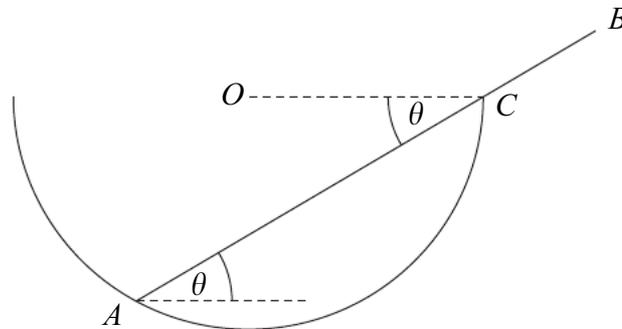
- (i) Find the extension of the string when the particle hangs in equilibrium. (2 marks)
- (ii) The particle is pulled down to a point A , which is 2.2 m below the point O . Calculate the elastic potential energy in the string. (3 marks)
- (iii) The particle is released when it is at rest at the point A .

Calculate the distance of the particle from the point A when its speed first reaches 0.8 m s^{-1} . (5 marks)

Turn over ►



- 9 A smooth hollow hemisphere, of radius a and centre O , is fixed so that its rim is in a horizontal plane. A smooth uniform rod AB , of mass m , is in equilibrium, with one end A resting on the inside of the hemisphere and the point C on the rod being in contact with the rim of the hemisphere. The rod, of length l , is inclined at an angle θ to the horizontal, as shown in the diagram.



- (a) Explain why the reaction between the rod and the hemisphere at point A acts through O . (1 mark)
- (b) Draw a diagram to show the forces acting on the rod. (2 marks)
- (c) Show that $l = \frac{4a \cos 2\theta}{\cos \theta}$. (5 marks)

